Random Number Generation for Excess Life of Mobile User Residence Time

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Abstract

In a mobile telecommunications network, the period when a mobile station (MS) resides in a cell (the radio coverage of a base station) is called the cell residence time of that MS. The period between when a call arrives at the MS and when the MS moves out the cell is called the excess life of the cell residence time for that MS. In performance evaluation of a mobile telecommunications network, it is important to derive the excess life distribution from the cell residence times. This distribution determines if a connected call will be handed over to a new cell, and therefore significantly affects the call dropping probability of the network. In mobile telecommunications network simulation, generating the excess-life random numbers is not a trivial task, which has not been addressed in the literature. This paper shows how to generate the random numbers from the excess life distribution, and develop the excess-life random number generation procedures for cell residence times with gamma, Pareto, lognormal and Weibull distributions. Our study indicates that the generated random numbers closely match the true excess-life distributions.

Keywords: Cell residence time, excess life, handover, mobility management

1 Introduction

A mobile telecommunications network is populated with several base stations (BSs). Mobile users receive mobile telecommunications services by using mobile stations (MSs) connecting to the BSs. When an MS moves from the radio coverage (called *cell*) of a BS to the radio coverage of another BS, the MS is disconnected from the old BS and re-connected to the new BS. This process is called *handover*. Figure 1 illustrates the relationship between movement of an MS and a call session to that MS. The MS moves to cell 1 at time t_0, and then moves to cell i at time t_i for $i > 1$. A call for the MS arrives at time t_1. If the call is not blocked or dropped, it completes at time t_5. At time t_1, if cell 1 does not have enough radio resources to accommodate this call (which can be a plain voice call or a multimedia call), the call is *blocked*. When the MS moves to cell i, the call is handed over from cell $i - 1$ to cell i. If no radio resources are available in cell i, the call is *dropped* or *forced to terminate*. Performance of a mobile telecommunications network is typically evaluated by the *call blocking probability* (a new call attempt is blocked), the *call dropping probability* or *force-termination* probability (a handover call is forced to terminate), and the *call incompletion probability* (a call is either blocked or dropped).

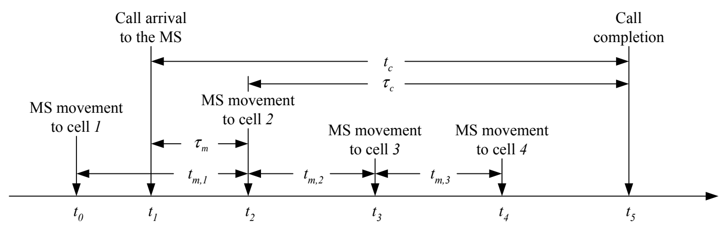

Figure 1: The Timing Diagram for MS Movement and Call Arrival

Many studies [3, 4, 9, 6, 15] have been devoted to evaluate these probabilities for various radio resource allocation strategies exercised in mobile telecommunications networks. Most of them utilized analytic approaches that provide useful insights to mobile network modeling. However, analytic analysis has its limitations. For example, in Figure 1, if the *call holding time* $t_c = t_5 - t_1$ is non-exponential (which is probably true for multimedia calls) [2], then it is difficult to derive the remaining call holding time $\tau_c = t_5 - t_i$ after the MS moves into cell i (for $i > 1$). Furthermore, most analytic studies made an approximate assumption that the handover traffic to a cell is

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a fixed Poisson Process. This assumption is reasonable for large-scale mobile telecommunications networks, but may result in significant inaccuracy for small-scale networks [16, 7]. Also, if the resource allocation policies under consideration are very complicate (which is probably true for wireless data sessions with QoS), it is impossible to find analytic solutions.

An alternative modeling technique to analytic analysis is discrete event simulation. There are two approaches to mobile telecommunications network simulation: the *MS-based* simulation and the *call-based* simulation. In the MS-based simulation, the number of MSs are defined in the simulation, and the MS objects are actually simulated for their movements (even if there are no calls destinated at these MSs). Examples of MS-based simulation can be found in [10]. In the call-based simulation [12, 8], the call arrival rate to the network is considered as the input that drives the simulation progress. In this approach, after a call arrival event is processed, the corresponding MS movement and the call termination events are generated following the timing diagram illustrated in Figure 1 (details of the call-based simulation is described in Appendix A). When the number of MSs is small in a mobile telecommunications network, the MS-based simulation will produce more accurate results than the call-based simulation. When the number of MSs is large, both approaches produce results with similar accuracies. On the other hand, the execution time for the MS-based simulation is much longer than that for the call-based simulation (e.g., 100 times longer [10]). Since large MS population is expected in most third generation systems such as UMTS (Universal Mobile Telecommunications System) [1, 11], the call-based simulation will become more important in advanced mobile telecommunications studies.

In mobile telecommunications network modeling, several random variables are defined. Two of them are elaborated here; others are described in Appendix A. In Figure 1, $t_{m,1} = t_2 - t_0$, and $t_{m,i} = t_{i+1} - t_i$ (for $i > 1$) are the time intervals that the MS resides in cell i. These *cell residence times* are typically modeled by a random variable with a specific distribution such as gamma and mixed Erlang [12, 8, 5]. The interval $\tau_m = t_2 - t_1$ is the period between when a call arrives and when the MS moves out of the first cell, which is referred to as the *excess life* of the cell residence time. In the call-based simulation, it is required to generate the random numbers for the excess life τ_m (see Appendix A). Clearly, the τ_m distribution must be derived from the cell residence time distribution. The call arrivals are typically assumed to be random observers of the cell residence times. If the cell residence times have the exponential distribution, then τ_m also has the same exponential distribution [14]. On the other hand, if the cell residence times have an arbitrary distribution, generation of the τ_m random numbers is a non-trivial task. In this paper, we describe how to generate the τ_m random numbers from the cell residence time distribution. For various cell residence time distributions, generation of τ_m random numbers need separate treatments. We show how to generate the excess-life random numbers for cell residence time random variables with gamma, Pareto, lognormal and Weibull distributions. Our study indicates that the generated random numbers closely match the true excess-life distributions.

2 Derivation of Excess Life Distribution

In Figure 1, the cell residence times $t_{m,i}$ $(i \geq 1)$ of an MS are assumed to be i.i.d. random variables. Therefore, we use t_m to represent an arbitrary cell residence time with the density function $f_m(t_m)$, the distribution function $F_m(t_m)$ and the mean μ. Let τ_m be the excess life of t_m with the density function $r_m(\tau_m)$ and the distribution function $R_m(\tau_m)$. Since the call arrivals form a Poisson process, a call arrival is a random observer of the MS cell residence times. From the excess life theorem [14], we have

$$r_m(\tau_m) = \frac{1 - F_m(\tau_m)}{\mu} \tag{1}$$

It is difficult to generate the random numbers for the excess life of a cell residence time random variable using Equation (1) because this equation involves the distribution function $F_m(\tau_m)$. To efficiently generate the random numbers τ_m, we shall utilize a variation of $f_m(\tau_m)$. We will prove that $r_m(\tau_m)$ can be derived from the following function

$$f_T(t) = \frac{t f_m(t)}{\mu} \tag{2}$$

Since

$$\int_{t=0}^{\infty} \left[\frac{t f_m(t)}{\mu} \right] dt = \left(\frac{1}{\mu} \right) \int_{t=0}^{\infty} t f_m(t) dt = \frac{\mu}{\mu} = 1$$

it is obvious that $f_T(t)$ can be a density function. Let T be a random variable with the density function $f_T(t)$. We have the following Theorem:

Theorem 1. Let τ_m be the excess life of t_m. Let random variable U be uniformly distributed over the interval $(0,1)$. Let T be random variable with the density function $f_T(t) = \frac{tf_m(t)}{\mu}$, and U and T are independent. Then the distribution of τ_m is the same as the distribution of $U \times T$.

Proof: The joint density function of U and T is

$$f_{(U,T)}(u,t) = \begin{cases} \frac{tf_m(t)}{\mu}, & \text{for } 0 < u < 1 \\ & \text{and } t > 0 \\ 0, & \text{otherwise} \end{cases}$$

Let $W = U \times T$. Then

$$\begin{aligned} \Pr[W \le w] &= \Pr[U \times T \le w] \\ &= \int_{u=0}^{1} \int_{t=0}^{\frac{w}{u}} f_{(U,T)}(u,t)\,dtdu \\ &= \int_{u=0}^{1} \int_{t=0}^{\frac{w}{u}} \frac{tf_m(t)}{\mu}\,dtdu \end{aligned} \quad (3)$$

From (3), the density function $f_W(w)$ of W can be derived as

$$\begin{aligned} f_W(w) &= \frac{d\Pr[W \le w]}{dw} \\ &= \int_{u=0}^{1} \left(\frac{w}{u}\right) \left[\frac{f_m(w/u)}{\mu}\right] \left(\frac{1}{u}\right) du \\ &= \left(\frac{1}{\mu}\right) \int_{u=0}^{1} \left(\frac{w}{u^2}\right) f_m(w/u)\,du \end{aligned} \quad (4)$$

Let $y = \frac{w}{u}$. Then (4) can be rewritten as

$$\begin{aligned} f_W(w) &= \frac{1}{\mu} \int_{y=w}^{\infty} f_m(y)\,dy \\ &= \frac{1 - F_m(w)}{\mu} \\ &= r_m(w) \end{aligned}$$

which means that $W = U \times T$ has the same distribution as τ_m.

Q.E.D.

Theorem 1 allows us to generate a τ_m random number using $f_m(\cdot)$ as follows: We first generate a random number u for the uniform random variable U in (0,1). Then we generate a random number t for the random variable T with the density function $f_T(t)$ (see (2)). Then we multiply t by u to obtain the random number for the excess life τ_m. Derivation of $f_T(t)$ is not a trivial task, and some $f_T(t)$ functions cannot not be derived from the corresponding $f_m(t)$ functions. In the next section, we show how to derive $f_T(t)$ for some popular distributions.

3 Excess-Life Random Number Generation: Some Examples

This section derives the T distributions for cell residence times with distributions such as gamma, Pareto, lognormal and Weibull. Then we show how to generate the excess-life random numbers using Theorem 1 and the T distributions.

3.1 The Gamma Distribution

Suppose that t_m has a gamma distribution with the shape parameter α and the scale parameter β. Then the mean value is $\mu = \alpha\beta$ and the density function $f_m(t_m)$ is

$$f_m(t_m) = \frac{e^{-\frac{t_m}{\beta}} t_m^{\alpha-1}}{\beta^\alpha \Gamma(\alpha)} \quad \text{for} \quad t_m \ge 0 \qquad (5)$$

We have the following theorem:

Theorem 2. If t_m has a gamma distribution with the parameters (α, β), then T has a gamma distribution with the parameters $(\alpha + 1, \beta)$.

Proof: From (2) and (5), we have

$$\begin{aligned} f_T(t) &= \frac{te^{-\frac{t}{\beta}} t^{\alpha-1}}{\mu\beta^\alpha \Gamma(\alpha)} \quad \text{for} \quad t \ge 0 \\ &= \frac{\beta e^{-\frac{t}{\beta}} t^{\alpha}}{\mu\beta^{\alpha+1} \Gamma(\alpha+1)} \times \frac{\Gamma(\alpha+1)}{\Gamma(\alpha)} \end{aligned} \quad (6)$$

Since $\Gamma(\alpha+1) = \alpha\Gamma(\alpha)$ and $\mu = \alpha\beta$, (6) is re-written as

$$f_T(t) = \frac{e^{-\frac{t}{\beta}} t^{\alpha}}{\beta^{\alpha+1} \Gamma(\alpha+1)} \quad \text{for} \quad t \ge 0 \qquad (7)$$

From (7), it is clear that T has the gamma distribution with parameters $(\alpha + 1, \beta)$.

Q.E.D.

Generation of an excess-life random number for gamma residence time with the parameters (α, β) includes the following steps: We first generate a uniform random number u in $(0,1)$. Then according to Theorem 2, we generate a random number t for the gamma random variable T with the parameters $(\alpha + 1, \beta)$. By multiplying u and t, we obtain a random number for the excess life τ_m. Figure 2 plots the $r_m(\tau_m)$ function for gamma excess life.

In this figure, the symbols "$\diamond$" and "$\bullet$" represent the values obtained from the random number generation. The solid and dashed curves are directly

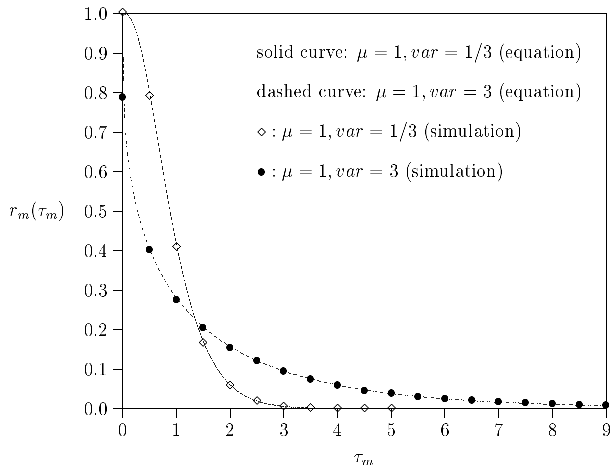

Figure 2: The $r_m(\tau_m)$ Function for Gamma Excess Life

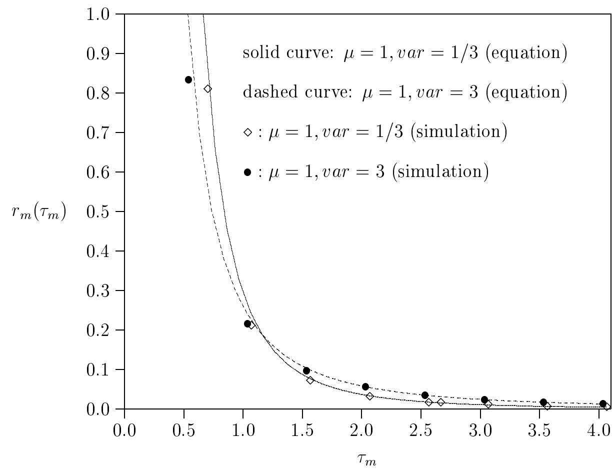

Figure 3: The $r_m(\tau_m)$ Function for Pareto Excess Life

computed from Equation (1). The figure indicates that our random number generation procedure accurately generates the excess-life random numbers for the gamma cell residence times.

3.2 The Pareto Distribution

Suppose that t_m has the Pareto distribution with the parameters (a, b), where a is the shape parameter and b is the scale parameter. Then the mean is

$$\mu = \begin{cases} \frac{ab}{a-1}, & \text{if } a > 1 \\ \infty, & \text{if } 0 < a \leq 1 \end{cases} \tag{8}$$

and the density function is

$$f_m(t_m) = \frac{ab^a}{t_m^{a+1}}, \tag{9}$$

where $t_m \geq b$, $a > 0$, and $b > 0$. We have the following theorem.

Theorem 3. Suppose that t_m has a Pareto distribution with the parameters (a, b), where $a > 1$. Then T has a Pareto distribution with the parameters $(a - 1, b)$.

Proof: From (2), (8) and (9), we have

$$\begin{aligned} f_T(t) &= \left(\frac{tab^a}{t^{a+1}}\right) \times \left(\frac{a-1}{ab}\right) \\ &= \frac{(a-1)b^{a-1}}{t^{(a-1)+1}} \end{aligned} \tag{10}$$

Equation (10) is a Pareto density function with the parameters $(a - 1, b)$.

Q.E.D.

By utilizing Theorems 1 and 3, the τ_m random number generation procedure for Pareto cell residence times is similar to that for gamma cell residence times. Figure 3 plots the $r_m(\tau_m)$ function for Pareto excess life. The figure indicates that our random number generation procedure accurately generates the excess-life random numbers for the Pareto cell residence times.

3.3 The Lognormal Distribution

Suppose that t_m has a lognormal distribution with the parameters (θ, σ). Then the mean value is $\mu = e^{\theta + \frac{\sigma^2}{2}}$ and the density function $f_m(t_m)$ is

$$f_m(t_m) = \left(\frac{1}{\sigma t_m \sqrt{2\pi}}\right) e^{-\frac{(\ln t_m - \theta)^2}{2\sigma^2}} \quad \text{for} \quad t_m \geq 0 \tag{11}$$

We have the following theorem.

Theorem 4. Suppose that t_m has a lognormal distribution with the parameters (θ, σ). Let $Y = \ln T$. Then Y has a normal distribution with the mean $\mu + \sigma^2$ and the standard deviation σ.

Proof: From (2) and (11),

$$\begin{aligned} f_T(t) &= \left(\frac{1}{\mu\sigma\sqrt{2\pi}}\right) e^{-\frac{(\ln t - \theta)^2}{2\sigma^2}} \quad \text{where} \quad t \geq 0 \\ &= \left(\frac{1}{e^{\theta + \frac{\sigma^2}{2}} \sigma \sqrt{2\pi}}\right) e^{-\frac{(\ln t - \theta)^2}{2\sigma^2}} \end{aligned} \tag{12}$$

Since $Y = \ln T$, we have $T = e^Y$. According to the Jacobian of the transformation [13], the density function of Y is expressed as

$$f_Y(y) = f_T(e^y)\left|\frac{dt}{dy}\right| = f_T(e^y) \times e^y \qquad (13)$$

where $-\infty < y < \infty$. Substitute (12) into (13) to yield

$$\begin{aligned} f_Y(y) &= \left(\frac{1}{e^{\theta+\frac{\sigma^2}{2}}\sigma\sqrt{2\pi}}\right) e^{-\frac{(y-\theta)^2}{2\sigma^2}} \times e^y \\ &= \left(\frac{1}{\sigma\sqrt{2\pi}}\right) e^{-\frac{[y-(\theta+\sigma^2)]^2}{2\sigma^2}} \qquad (14) \end{aligned}$$

where $-\infty < y < \infty$. From (14), Y is a normal random variable with the mean $\theta + \sigma^2$ and the standard deviation σ.

Q.E.D.

Generation of an excess-life random number for lognormal cell residence time with the parameters (θ, σ) includes the following steps: We first generate a random number u from the uniform random variable U in $(0, 1)$. Then according to Theorem 4, we generate a random number y for the normal random variable Y with the mean $\theta + \sigma^2$ and the standard deviation σ. By multiplying u and e^y, we obtain a random number for the excess life τ_m. Details of the lognormal residence time curves will not be presented in this paper.

3.4 The Weibull Distribution

Suppose that t_m has a Weibull distribution with the scale parameter θ and the shape parameter γ. Then the mean value is $\mu = \theta^{\frac{1}{\gamma}}\Gamma\left(1 + \frac{1}{\gamma}\right)$ and the density function $f_m(t_m)$ is

$$f_m(t_m) = \begin{cases} \left(\frac{\gamma}{\theta}\right) t_m^{\gamma-1} e^{-\frac{t_m^\gamma}{\theta}}, & \text{if } t_m \ge 0, \\ 0, & \text{if } t_m < 0 \end{cases} \qquad (15)$$

We have the following theorem.

Theorem 5. Suppose that t_m has a Weibull distribution with the parameters (γ, θ). Let $Y = T^\gamma$. Then Y has a gamma distribution with the parameters $\left(1 + \frac{1}{\gamma}, \theta\right)$.

Proof: From (2) and (15), we have

$$f_T(t) = \left(\frac{\gamma}{\theta}\right) \times \left[\frac{t^\gamma e^{-\frac{t^\gamma}{\theta}}}{\theta^{\frac{1}{\gamma}}\Gamma\left(1 + \frac{1}{\gamma}\right)}\right] \qquad (16)$$

where $t \ge 0$. Let $Y = T^\gamma$. Then $T = Y^{\frac{1}{\gamma}}$. According to the Jacobian of the transformation [13], the density function of Y is

$$f_Y(y) = f_T\left(y^{\frac{1}{\gamma}}\right)\left|\frac{dt}{dy}\right| = f_T\left(y^{\frac{1}{\gamma}}\right) \times \left(\frac{y^{\frac{1}{\gamma}-1}}{\gamma}\right) \qquad (17)$$

where $y \ge 0$. Substitute (16) into (17) to yield

$$f_Y(y) = \frac{y^{\frac{1}{\gamma}} e^{-\frac{y}{\theta}}}{\theta^{1+\frac{1}{\gamma}}\Gamma\left(1 + \frac{1}{\gamma}\right)} \quad \text{where} \quad y \ge 0 \qquad (18)$$

From (18), Y has a gamma distribution with the parameters $\left(1 + \frac{1}{\gamma}, \theta\right)$.

Q.E.D.

Generation of an excess-life random number for Weibull cell residence time with the parameters (γ, θ) includes the following steps: We first generate a random number u of the uniform random variable U in $(0, 1)$. Then according to Theorem 5, we generate a random number y for the gamma random variable Y with the parameters $\left(1 + \frac{1}{\gamma}, \theta\right)$. By multiplying u and $y^{\frac{1}{\gamma}}$, we obtain a random number for the excess life τ_m. Details of the Weibull residence time curves will not be presented in this paper.

4 Conclusions

In performance evaluation of a mobile telecommunications network, it is important to derive the excess life distribution from the cell residence times. This distribution determines if a connected call will be handed over to a new cell, and therefore significantly affects the call dropping probability of the network. In mobile telecommunications network simulation, generating the excess-life random numbers is not a trivial task, which has not been addressed in the literature. This paper showed how to derive the excess life distribution and to generate the random numbers from the excess life distribution. We then developed the excess-life random number generation procedures for cell residence times with gamma, Pareto, lognormal and Weibull distributions. Our study indicates that the generated random numbers closely match the true excess-life distribution (i.e., Equation (1)). Therefore our procedures can be utilized to efficiently generate excess-life random numbers in mobile telecommunications network simulation.

A The Call-based Simulation

This appendix describes the basic call-based discrete event simulation for mobile telecommunications network. Several random variables are defined: the *inter call arrival time* (the call arrivals are typically modeled as a Poisson process), the *call holding time*, the *cell residence time* and the *excess life* of the cell residence time. Three basic event types are considered: the **arrival** event (a call arrival), the **move** event (an MS movement), and the **complete** event (a call completion). Every event is associated with a timestamp representing the time when the event occurs. All unprocessed events are inserted in an *event list* and are processed in the non-decreasing timestamp order. Details of the call-based simulation are described in the following steps:

Step 1 (Initialization): Generate the first **arrival** event and insert it in the event list.

Step 2. Remove the next event from the event list. If the event type is **arrival** then go to Step 3. If the type is **move** then go to Step 5. If the type is **complete** then go to Step 6.

Step 3 (arrival). Check if the cell can accommodate this call based on some wireless resource allocation policy. If not, reject the call, update the call statistics, and go to Step 4. Otherwise, generate the random numbers for the excess life τ_m of the cell residence time and the call holding time t_c.

Step 3.1. If $\tau_m > t_c$, generate a **complete** event with timestamp "current time+t_c".

Step 3.2. If $\tau_m < t_c$, generate a **move** event with timestamp "current time+τ_m". Note that when the next **move** event occurs, the remaining call holding time is $\tau_c = t_c - \tau_m$.

Insert the generated event into the event list.

Step 4. Generate the next **arrival** event according to the Poisson process and insert it into the event list. Go to Step 2.

Step 5 (move). The MS moves from the old cell to the new cell. Check if the new cell can accommodate this handover call. If not, drop the call, update the call statistics, and go to Step 2. Otherwise, generate the cell residence time t_m. The remaining call holding time is τ_c.

Step 5.1. If $t_m > \tau_c$, generate a **complete** event with timestamp "current time+τ_c".

Step 5.2. If $t_m < \tau_c$, generate the next **move** event with timestamp "current time+t_m". Note that when the next **move** event occurs the remaining call holding time is $\tau_c = \tau_c - t_m$.

Insert the generated event into the event list. Go to Step 2.

Step 6 (complete). Reclaim the resources used by this call. Update the call statistics, and go to Step 2.

The simulation can be terminated based on various criteria. For example, at Step 3, we may check if some terminating conditions are satisfied (e.g., 1000,000 call arrivals have been simulated). If so, the simulation terminates.

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