FishTalk: An IoT-based Mini Aquarium System
Yi-Bing Lin†, Fellow, IEEE, and Hung-Chun Tseng†
†Department of Computer Science National Chiao Tung University, Hsinchu 30010, Taiwan, ROC

ABSTRACT Many people feed the pet fish in the aquarium tanks that need to be properly set up and maintained, or the fish will be destined to unpleasant and short life. Therefore it is critical to monitor water conditions closely and improve the water quality for the mini aquarium tanks. Based on an IoT solution called IoTtalk, this paper proposes the FishTalk system that utilizes the aquarium sensors to drive the actuators in real time. We describe the relationship between aquarium sensors and the actuators, and give concrete examples about threshold setting. Our solution allows the designer to quickly deploy intelligent control for various water conditions. As an example we implement an intelligent fish feeding mechanism such that the fish are neither over nor under fed, and at the same time, the fish owner can enjoy watching fish feeding remotely. We have also developed analytic model, simulation and measurement experiments to investigate the effects of IoT message delays and loss on water condition control.

INDEX TERMS Aquarium, Internet of Things (IoT), message lost, NB-IoT, performance evaluation

I. INTRODUCTION
In the recent years, mini aquarium tanks in various shapes and sizes are replacing classic goldfish bowls for small apartments or college dorm rooms. For example, college students in a dorm, where no-pet rules do not apply to fish, have space for a mini aquarium. At National Chiao Tung University (NCTU), we encourage students to watch fish for relaxation and to care for them in the dorms where larger pets are not allowed. However, the aquarium tanks need to be properly set up and maintained, or the fish will be destined to unpleasant and short life. Specifically, when the water volume in the tank is small, key water parameters change very quickly, leaving no room for error. Therefore it is critical to monitor water conditions closely and perform water changes faithfully for the mini aquarium tanks. Many commercial aquarium tanks under one gallon claim that no maintenance is required. However, such products are actually not healthy for fish, and therefore are not recommended. Several studies [1], [2] indicate that small aquarium tanks are not suitable for maintaining healthy fish. The bigger the tank, the less impact a mistake will have on fish, and aquarium tanks of at least five gallons are suggested. In this paper, we consider mini tanks with sizes larger than 60×45×60 cm³. Even so, key water parameters of a mini aquarium tank should be closely watched, and proper actions should be quickly taken when some dangerous situations are detected. Since student dorms are often empty during class hours, no one may be able to detect the abnormal situations of the tank. Therefore, several solutions based on Internet of Things (IoT) [3],[4],[5],[6],[7],[22],[23] were proposed to automate the detection of abnormal aquarium situations. However, few of them have provided intelligent mechanisms to automatically activate the actuators to fix the problems. In NCTU, we have developed an IoT solution called IoTtalk for smart dorm, smart garden and other smart campus applications [8][24]. Based on IoTtalk, this paper proposes the FishTalk system that allows the aquarium sensors to drive the actuators in real time. Table 1 compares FishTalk with the previous solutions. In item 1 of the table, FishTalk uses more sensors than the previous solutions except for [23]. In item 2, FishTalk accommodates more actuators than the previous solutions. Item 3 indicates that solutions [3][7][22][23] provided sensor monitoring but no actuator control. Solutions [1][4] provided manual control of actuators but cannot automatically control the actuators by the sensors. Solution [6] provides trivial actuator control based on simple sensor thresholds. On the other hand, FishTalk provides both manual control and automatic actuator control by the sensors with non-trivial intelligence other than simple “threshold control”. This intelligent feature allows much better control of the water environment. Also, through the time series charts of sensors/actuators in FishTalk’s display (to be elaborated in Section V), the fish owner can easily learn how
the water factors interact with each other. In this way, the sensors do not just provide raw data (as the previous solutions do) but also allow the fish owners to know more about their aquarium environments. We have developed this type of FishTalk applications to be used by National Taiwan Science Education Center to teach students in junior high schools for aquarium science experiments.

Overfeeding is the number one mistake made by fish owners, as uneaten food will pollute the water. In item 4, none of previous solutions provide fish feeding mechanism except for [6] that uses a mechanical timer to automatically trigger the feeding mechanism. Such mechanism is not reliable and may drop too much food to kill the fish. With FishTalk, one can easily implement smart feeding that allows fish owner to remotely enjoy manual feeding while the fish are neither underfed nor overfed (to be described in Section V).

In item 5, FishTalk provides video monitoring that are not found in other smart aquarium solutions. The cost of off-the-shelf camera can be shared by other smart applications in a room. In [24] we have designed a smart saloon in a student dormitory where one camera is used to remotely monitor and enjoy the views of various IoT applications (such as smart dart, curtain control, smart plant and FishTalk).

To simplify and strengthen our discussion, we focus on freshwater tank without plants. The paper is organized as follows. Section II introduces the dissolved gases that affect the water quality of the aquarium tank. Sections III and IV describe the sensors and the actuators used in FishTalk, respectively. Section V proposes FishTalk as an IoT-based aquarium system. Based on NB-IoT and Wi-Fi, Section VI conducts performance evaluation on the IoT message delays and their impact on the aquarium operations. Section VII concludes our work by listing three major contributions not found in the previous studies. This paper is written for the IoT experts who do not have aquarium knowledge. For the reader who knows aquarium well, Sections II-IV can be skipped.

II. DISSOLVED GAS FACTORS FOR WATER QUALITY

Dissolved gases such as carbon dioxide, oxygen and ammonia greatly influence the water quality in an aquarium tank. These gases are described in this section.

A. CARBON DIOXIDE

Dissolved carbon dioxide causes suffocation and lowers the pH of water, which leads to stressful hypercapnia conditions for fish and will eventually result in death. When the water tank has high levels of free carbon dioxide, fish have to adjust their blood bicarbonate levels to avoid acidosis, which can happen when the transfer of carbon dioxide from the fish’s blood to the surrounding water is greatly reduced. When the carbon dioxide level in water is high, fish exposed to hypercapnia can recover by significantly increasing their blood plasma and taking up bicarbonate in exchange for chloride. However, when high carbon dioxide levels are coupled with low oxygen levels, the result is a decrease in oxygen affinity and sometimes carrying capacity of the blood, which is often fatal. Although the toxicity threshold for free carbon dioxide concentration can vary considerably, depending on dissolved oxygen levels, a free carbon dioxide concentration of 30 parts per million (ppm) or milligrams per liter (mg/L) is typically safe for most fish [9].

B. OXYGEN

An aquarium needs oxygen to support the livestock. Decreased oxygen concentration combined with elevated carbon dioxide concentration in the water leads to suffocation [10]. The oxygen requirements differ depending on the type and the weight of fish. For some species without additional organs, the result of low dissolved oxygen is immediate death. Some species have developed labyrinth organs to allow air breathing in hypoxic conditions. However, oxygen deprivation (hypoxia) still occurs when the oxygen concentration in the water is lower than the fish's minimum requirements. The pH of water, which leads to stressful hypercapnia conditions for fish and will eventually result in death. When the water tank has high levels of free carbon dioxide, fish have to adjust their blood bicarbonate levels to avoid acidosis, which can happen when the transfer of carbon dioxide from the fish’s blood to the surrounding water is greatly reduced. When the carbon dioxide level in water is high, fish exposed to hypercapnia can recover by significantly increasing their blood plasma and taking up bicarbonate in exchange for chloride. However, when high carbon dioxide levels are coupled with low oxygen levels, the result is a decrease in oxygen affinity and sometimes carrying capacity of the blood, which is often fatal. Although the toxicity threshold for free carbon dioxide concentration can vary considerably, depending on dissolved oxygen levels, a free carbon dioxide concentration of 30 parts per million (ppm) or milligrams per liter (mg/L) is typically safe for most fish [9].

Table 1. The IoT solutions for Smart Aquarium

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2. Actuators</td>
<td>Heater</td>
<td>No</td>
<td>Air Pump</td>
<td>Heater, Light Feeder, Air Pump.</td>
<td>No</td>
<td>No</td>
<td>Light</td>
<td>Feeder, Fan, Heater, Light, Air Pump, RO Filter</td>
</tr>
<tr>
<td>3. Actuators Controlled by Sensors</td>
<td>No (Manual)</td>
<td>No</td>
<td>No (Manual)</td>
<td>Yes (Simple threshold)</td>
<td>No</td>
<td>No</td>
<td>No (Light is always on at night)</td>
<td>Yes</td>
</tr>
<tr>
<td>4. Smart Feeder</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No (manual)</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>5. Video Monitoring</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>6. Control Board</td>
<td>No</td>
<td>N/A</td>
<td>Raspberry Pi 3</td>
<td>MSP430</td>
<td>Arduino Uno</td>
<td>Arduino Uno</td>
<td>Raspberry Pi 3</td>
<td>Arduino UNO, ESP8266 ESP-12F, ROHM IoT kit, MediaTek LinkIt Smart 7688 Duo</td>
</tr>
</tbody>
</table>

The IoT solutions for Smart Aquarium

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2. Actuators</td>
<td>Heater</td>
<td>No</td>
<td>Air Pump</td>
<td>Heater, Light Feeder, Air Pump.</td>
<td>No</td>
<td>No</td>
<td>Light</td>
<td>Feeder, Fan, Heater, Light, Air Pump, RO Filter</td>
</tr>
<tr>
<td>3. Actuators Controlled by Sensors</td>
<td>No (Manual)</td>
<td>No</td>
<td>No (Manual)</td>
<td>Yes (Simple threshold)</td>
<td>No</td>
<td>No</td>
<td>No (Light is always on at night)</td>
<td>Yes</td>
</tr>
<tr>
<td>4. Smart Feeder</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No (manual)</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>5. Video Monitoring</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>6. Control Board</td>
<td>No</td>
<td>N/A</td>
<td>Raspberry Pi 3</td>
<td>MSP430</td>
<td>Arduino Uno</td>
<td>Arduino Uno</td>
<td>Raspberry Pi 3</td>
<td>Arduino UNO, ESP8266 ESP-12F, ROHM IoT kit, MediaTek LinkIt Smart 7688 Duo</td>
</tr>
</tbody>
</table>
tension is lower than the organism’s requirement. Depending on the fish species, the oxygen levels should be higher than a certain concentration (usually 2-4 mg/L) to avoid oxygen depletion.

C. AMMONIA
Free Ammonia is extremely toxic to fish [1]. High ammonia concentration can cause a decrease in blood serum ATP and lead to tissue necrosis. This further increases the energy demands on the gill organism. Initially, the fish might appear to gasp at the surface for air while their gills take on a red or lilac color. Then the fish might lie at the bottom of the tank with clamped fins as their body functions fail. The fish become increasingly lethargic and start losing their appetites. The brain, organs, and the central nervous system become damaged. Finally, the fish begin to hemorrhage, and eventually die.

Chemically treated tap water and the decomposition of organic matter inside the tank (e.g., aquarium plants, fish excrement, and uneaten fish food) contribute to ammonia poisoning (free ammonia; to be elaborated in Section III-B) that often occurs during the setup of a new tank. It can also occur when too many fish are added at one time, or when filters are not kept clean. Ammonia poisoning also occurs if the water is not changed regularly or if bacterial colonies die off due to a sudden change in water conditions or the use of medications.

III. THE AQUARIUM SENSORS
To control the dissolved gases and other factors that affect the health of fish, we propose an IoT solution called FishTalk. Several sensors have been deployed in FishTalk including e.g., the dissolved oxygen (DO), the temperature, the pH, the electrical conductivity (EC), the water level, and the total dissolved solids (TDS) sensors illustrated in Figure 1. The sensors send the measured data to FishTalk either periodically or when some events occur.

A. THE TEMPERATURE SENSORS
Studies indicate that higher temperatures within the optimal temperature range of the species typically leads to healthier fish with stronger immune functions [11], [12]. For most fishes, the optimal temperature ranges from 25° to 27°C. Extreme changes in temperature are more harmful to fish than constant high or low temperatures.

The water temperature also interacts with factors such as elevated ammonia and decreased oxygen. For example, the amount of oxygen that can be dissolved in the water depends on the water temperature and salinity levels. Decreasing temperature and salinity in the water results in increased oxygen saturation [13]. Figure 2 plots dissolved oxygen (in mg/L) against water temperatures in centigrade and salinity levels in parts per thousand (ppt), where salinity of 0 corresponds to freshwater. This figure shows the 100% saturation values for dissolved oxygen. The average saturation in an aquarium tank is about 70% of the values shown in the figure. Figure 1 (b) illustrates the temperature sensor used in FishTalk.

B. THE PH SENSOR
The dissolved carbon dioxide creates carbonic acid, which acidifies the water that can be measured by the pH sensor (Figure 1 (c)). Water pH is affected by water hardness (see Section III-C), fish and plant waste, topping off the water and water evaporation, and sudden changes in the water results in changes in blood pH, which leads to stress and death. The comfortable pH level ranges from 6.5 to 9. Specifically, the preferred pH range is 6.5 - 7.0 for angel loach, harlequin and tiger barb. The preferred pH range is 5.8 - 6.2 for neon tetra, 6.5-7.0 for zebra danio and 7.0-7.5 for goldfish.

Low pH (e.g., less than 5.5) ceases the proton movement from the fish body. On the other hand, high pH increases the free ammonia level. Note that ammonia exists in aquaria in
two forms: free ammonia (NH$_3$) and ammonium, where free ammonia is the toxic part. Figure 3 illustrates that for a fixed amount of ammonia (e.g., 2.0 ppm), the portion of free ammonia in the tank water is affected by temperature and pH. Specifically, free ammonia increases as temperature and pH increase [14]. When the free ammonia level is higher than 0.02 ppm, the fish owner should be cautious, and the water must be changed when the level is higher than 0.05 ppm.

\[ \text{FIGURE 3. Effects of temperature and pH on free ammonia.} \]

C. THE ELECTRICAL CONDUCTIVITY SENSOR AND CARBONATE HARDNESS

Another important aspect of water chemistry is carbonate hardness (KH) that represents the pH buffering capacity of the water. Harder water will have a higher buffering capacity. The ideal KH medium range is 4°–6°KH. However, the KH ranges are much higher for specific species, such as poeciliidae (8°–10°KH), African cichlid (±20°KH), and salt water fishes (10°–15°KH). In [9], water with a KH of 4° (4KH) is suggested to monitor the CO$_2$ level with more accuracy.

EC is roughly related to general hardness (GH) and TDS [15], and a correct EC level is needed to keep the fish healthy. For example, comfortable EC value ranges from 100 to 300 µS/cm for community freshwater tanks. For the Discus and Paracheirodon species, the required EC values are below 100µS/cm, while cichlids from African lakes (i.e., Malawi and Tanganyika) grow well at values above 500 µS/cm.

Pollutants affect the EC value by increasing it. Therefore, a change in the EC value indicates a change in water conditions. The appropriate EC value can be easily maintained by changing the water to reduce the pollutants. If the EC value is too high, an appropriate amount of osmotic water (see Section IV-D) can be used to dilute the tank water. On the other hand, if the EC value is too low for the species (i.e. African cichlids), then the water needs to be hardened by using some commercial salt mixes or calcium carbonate solutions. Figure 1 (d) illustrates the EC sensor used in FishTalk.

IV. THE AQUARIUM ACTUATORS

The FishTalk sensors measure the aquarium environment and indicate how and when to maintain the tank. If we fail to maintain the aquarium tank, the fish will be stressed by deficient water conditions, will be more susceptible to disease, and often will have a shorter lifespan. In FishTalk, maintenance, with the exception of changing water, is automatically performed by the aquarium actuators. In the current version of FishTalk, water is changed manually or semi-automatically with the assistance of the water pumps.

A water level sensor (Figure 1 (e)) is used to detect the water level of the aquarium tank. When the water level is too low (e.g., caused by evaporation of fan blowing described in Section IV-B), FishTalk can remind the fish owner when to change/add water through an alert mechanism. The frequency varies depending on many factors. For a large, sparsely stocked aquarium, 10 to 15 percent of the water should be changed every two weeks. For a small, heavily stocked tank, up to 20 percent of the water should be changed each week. For a lazy fish owner, as long as he/she is regularly performing partial water changes every couple of weeks, the exact frequency is not so critical. The actuators automatically activated by FishTalk are described in the remainder of this section.

\[ \text{FIGURE 4. The FishTalk actuators (RO: Reverse Osmosis).} \]

A. FOOD FEEDER

Fish are opportunistic and will seek food at all times. Overfeeding is the number one mistake made by fish owners, as uneaten food will pollute the water. In fact, fish can easily go several days without food, and there are few ill effects for
slight underfeeding. When free ammonia is being built up (e.g., during the setup of a new tank), it is advised to feed fish no more than once per day to reduce the waste productions. In FishTalk, a food feeder (Figure 4 (a)) is smartly and automatically controlled. The details will be given in the next section.

B. AQUARIUM HEATER AND FAN
A heater (Figure 4 (b)) is used to increase the water temperature in an aquarium tank. Unfortunately, mini tanks are difficult to heat properly, and heaters specifically designed for mini aquariums should be selected. To disperse heated water more quietly and evenly throughout the tank, the heater should be placed near the filter intake so that the warm water from the heater will be sucked through the filter’s intake along with a cooler stream from the bottom of the aquarium.

To reduce the water temperature, a fan (Figure 4 (c)) can be used to blow across the surface of the tank water to increase evaporation. Water releases much more energy when it transitions from water to vapor, which drops the temperature. Therefore, a fan blowing across the surface of the tank water reduces water temperature by increasing evaporation.

C. WATER PUMP AND LIGHT
The water pump (Figure 4 (d)) generates currents and aeration, and move water through aquarium tank peripherals such as the filters. Water movement creates aeration by constantly mixing the surface with the rest of the water. The currents prevent detritus from accumulating. It is suggested to select the pump capable of moving five times as many gallons per hour as the tank holds.

If live plants, photosynthetic invertebrates, or macroalgae are not kept in an aquarium tank, the only lighting needed on the tank is for viewing fish. Light does not penetrate water easily, and every inch of depth greatly decreases the effective illumination. The use of good reflectors can increase effective illumination by directing more of the light produced into the tank. Very high output (VHO) or power compact (PC) lamps can be used for a 12-inch-deep tank (Figure 4 (e)). For a depth of 18 or 24 inches, metal halides are more appropriate.

D. REVERSE OSMOSIS FILTER
The Reverse Osmosis (RO; see Figure 4 (f)) lowers pH in the aquarium tank and purifies the tap water. An efficient RO unit can remove 90% or more of tap water contaminants, the semi-permeable membrane acts as an ultra-fine filter to allow only water molecules to pass through, and strain most unwanted constituents. This part is expensive and easily damaged, and some high-output units may have multiple membranes. The pre-filters are placed before the membrane to remove sediment, chlorine and other components of mains water which would rapidly block and/or destroy the membrane. A flow restrictor is a valve that allows pressure to build up in the system for reverse osmosis to take place. A flush valve may be incorporated to bypass the flow restrictor. Therefore, the deposits can be washed from the membrane to improve the RO efficiency [16].

Although the RO filter is efficient at removing unwanted ions from water, the RO membranes may become blocked over time. Such a problem cannot be confirmed by visual inspection, but can be detected by the TDS sensor (Figure 1 (f)) if its value is high. The TDS sensor detects anions and cations (such as magnesium, calcium, silicate, sodium, phosphate and nitrate) in the water of the aquarium tank. When the TDS value is higher than e.g., 10 ppm, it is an early warning for the deterioration of tank water. Note that the TDS sensor does not work for reef aquaria with salt water.

The waste water produced from the RO outlet has elevated levels of nitrates, phosphates, heavy metals, and pesticides that should not exist in the aquarium tank. This waste water is reused for garden plants in the student dorms at NCTU.

V. CREATION OF FISHTALK PROJECTS
FishTalk is an IoT-based aquarium system developed according to an IoT device management platform called IoTtalk [8][17][18]. Figure 5 illustrates a simplified block diagram for FishTalk, which consists of the FishTalk sensors (Figure 5 (1)), actuators (Figure 5 (2)) and the FishTalk server (Figure 5 (3)).

![Figure 5: A simplified block diagram for FishTalk.](image)

In FishTalk, the IoT devices can connect to the FishTalk server through various communications technologies such as NB-IoT, LoRa, Sigfox, Wi-Fi, LTE and Ethernet. For the example in Figure 5, the sensors are connected to a NB-IoT-based control board (Figure 5 (4)) [19]. This control board interacts with the FishTalk server through NB-IoT wireless communications (Figure 5 (5)). Similarly, the actuators are instructed by the FishTalk server through another NB-IoT based control board (Figure 5 (6)). Besides the sensors, any standard smartphone (Figure 5 (7)) can connect to the FishTalk server through its browser based on the LTE or the 5G technologies for video monitoring (Figure 5 (8)). Note that the NB-IoT based control boards are typically used in outdoor environments. In an indoor environment, Wi-Fi based control boards are used [20].

The FishTalk web page of the smartphone includes four areas. The control sliding bar area (Figure 6 (1)) provides soft switches to control the heater, the fans and other
actuators connected to FishTalk. The display bar area (Figure 6 (2)) shows the real-time values of the sensors connected to FishTalk. The video control bar area (Figure 6 (3)) provides buttons to zoom in, zoom out and rotate a camera for viewing the aquarium tank (Figure 6 (4)).

FIGURE 6. The smartphone browser for FishTalk.

When an icon in the display bar area is clicked, the smartphone shows the time series chart of the sensor as illustrated in Figure 7. In this figure, we turn on the fans at point (1). Then we turn on the heater at point (2) and then turn it off shortly. The time series charts indicate that from point (1) to point (2), the temperature decreases, and the EC value increases. At point (2), the temperature increases sharply and then decreases again. The example in Figure 7 shows that the FishTalk web-based display provides chart information that allows the fish owner to understand the aquarium water conditions easily.

FIGURE 7. Time series of the sensor values.

FishTalk provides a friendly web-based graphical user interface (GUI) to allow quick development of IoT applications. Figure 8 shows the web window for the FishTalk project “Aquarium” (Figure 8 (1)). In the “Model” pulldown menu bar (Figure 8 (2)), the FishTalk sensors, controls, and the actuators can be selected, and the icons of the selected items are shown in the window. The sensors icon (Figure 8 (3)) includes temperature, pH, EC, DO and TDS. The actuators icon (Figure 8 (4)) includes food feeder, pump, heater, fan, light, UV and RO. The controls icon (Figure 8 (5)) includes the “on/off” switches for the actuators. It is very easy to connect a soft control switch to the corresponding actuator: simply drag a line between the switch icon to the actuator icon. For example, the link “Join 5” allows the “FoodFeeder-I” switch to control the food feeder. When the link is created, the FishTalk engine automatically generates a program to handle the interaction between the control and the actuator. Similarly, an actuator can be controlled by multiple sensors through the Join links. In our configuration, the heater is controlled by the temperature sensor through Join 1. The fans are controlled by both the temperature and the pH sensors through Join 2. The pump is controlled by the temperature and the DO sensors through Join 3. The RO filter is controlled by the EC and the TDS sensor through Join 4.

FIGURE 8. Configuration of the Aquarium project.

A Join connection is decomposed into two segments and a circle. The circle represents a function implemented in Python. The first segment connects one or more sensors/actuators to a circle, which provides the paths to deliver the data to the circle (the inputs of the Python function). The second segment connects the circle to one or more actuators, which sends the instruction made at the circle to the connected actuators (the output of the function). To write a Python function, one simply clicks the circle, and the FishTalk GUI pops up a window (Figure 9). For Join 1 in Figure 8, a heater control function is implemented as follows (Lines 1-5 in Figure 9). The input argument *args of the Python function run() stores the temperature value from
Temp-i (i.e., args[0]). If the temperature is higher than 27°C, then Join 1 returns the output “0” (turn-off) to Heater-O. If the temperature is lower than 23°C, Join 1 returns “1” (turn-on). When the temperature is in the range [23°C, 27°C], no action is taken (i.e., the server does not send any output to Heater-O).

**FIGURE 9.** The heater control function.

Based on the above description, the FishTalk GUI is a powerful tool that allows the designer to quickly deploy a FishTalk project. The graphical configuration allows one to easily understand how the actuators are controlled, and the project is convenient to maintain and debug. The graphical configuration together with the FishTalk time series charts allow one to trace complicated ripple through interactions. For example, one may observe that when the fans are turned on, the RO filter is also turned on after a short period of time. In Figure 8, the fan is turned on either through Join 2 or Join 8. From the time series charts in Figure 7, one observes that the temperature decreases (due to fan blowing), and the EC increases. From the graphical configuration in Figure 8, one immediately finds that when the EC value reaches a threshold, FishTalk triggers the RO filter through Join 4.

We can create multiple FishTalk projects for an aquarium tank. Besides the Aquarium project, we may create another project “FishFeeder” to control the food feeder (Figure 10 (1)). In this project, the fish are automatically fed for, say, every 24 hours. The fish owner and his/her guests are allowed to trigger the food feeder through a smartphone if the fish have not been fed for at least 12 hours. The FishFeeder project guarantees that the time period between two meals is longer than 12 hours and is shorter than 24 hours. We use a cyber FishTalk IoT device “Timer” to implement the mechanism to guarantee that the fish will have a meal within 24 hours. This device includes a control CountDown-I (Figure 10 (2)) and an actuator Reset-O (Figure 10 (3)).

**FIGURE 10.** The FishFeeder project.

The Python function for Join 1 in Figure 10 is illustrated in Lines 1-12 in Figure 11. The function uses the system time “clock” (Line 5). Through *args, the timer value of CountDown-I is assigned to count_down (Line 2), the button pressed time of FeedTime-I is assigned to feed_time (Line 3) and the TDS value of TDS-I is assigned to tds (Line 4). If tds is less than 600 ppm, then FoodFeeder-O is activated (for one meal) in two cases: either count_down is 0 (Line 8) or if the feed button is pressed at the current time (feed_time == clock; see Line 11) when count_down is less than 12 hours (Line 10). If tds>600 or none of the conditions in Lines 8 and 10 are met, then Join 1 does not produce any output. Otherwise, Join 1 will output the non-zero signal to instruct the Timer device to reset the countdown timer to 24 hours again. The food feeder (Figure 10 (5)) is controlled by CountDown-I and the soft button FeedButton in a smartphone (Fig, 10 (4)). In the FeedButton device, when one presses the button, FeedTime-I sends out the time when the button is pressed. Furthermore, if the water quality is poor (TDS is higher than 600 ppm) the feeding mechanism is disabled. Without loss of generality, the FishFeeder project uses the TDS sensor (TDS-I; see Figure 10 (6)) to monitor the water quality. Other sensors can be included in the Sensors device for water quality monitoring. TDS-I is also connected to a cyber device Messaging (Figure 10 (7)) through Join 2. When the water quality is poor, the fish owner will receive a warning message from WQ-Alert-O.

The **Python function for Join 1** in Figure 10 is illustrated in Lines 1-12 in Figure 11. The function uses the system time “clock” (Line 5). Through *args, the timer value of CountDown-I is assigned to count_down (Line 2), the button pressed time of FeedTime-I is assigned to feed_time (Line 3) and the TDS value of TDS-I is assigned to tds (Line 4). If tds is less than 600 ppm, then FoodFeeder-O is activated (for one meal) in two cases: either count_down is 0 (Line 8) or if the feed button is pressed at the current time (feed_time == clock; see Line 11) when count_down is less than 12 hours (Line 10). If tds>600 or none of the conditions in Lines 8 and 10 are met, then Join 1 does not produce any output. Otherwise, Join 1 will output the non-zero signal to instruct the Timer device to reset the countdown timer to 24 hours again. The food feeder (Figure 10 (5)) is controlled by CountDown-I and the soft button FeedButton in a smartphone (Fig, 10 (4)). In the FeedButton device, when one presses the button, FeedTime-I sends out the time when the button is pressed. Furthermore, if the water quality is poor (TDS is higher than 600 ppm) the feeding mechanism is disabled. Without loss of generality, the FishFeeder project uses the TDS sensor (TDS-I; see Figure 10 (6)) to monitor the water quality. Other sensors can be included in the Sensors device for water quality monitoring. TDS-I is also connected to a cyber device Messaging (Figure 10 (7)) through Join 2. When the water quality is poor, the fish owner will receive a warning message from WQ-Alert-O.
def run(*args):
    Line1 import datetime
    Line2 count_down = args[0][0][0]
    Line3 feed_time = args[0][1][0]
    Line4 tds = args[0][2][0]
    Line5 clock = (datetime.datetime.now())
    Line7 if tds < 600:
        Line8 if count_down == 0:
            Line9 return 1
            Line10 else if (count_down < (3600*12):
                Line11 and feed_time == clock):
                    Line12 return 1

FIGURE 11. The food feeder control function.

Note that if both the Aquarium and the FoodFeeder projects are enabled, then the food feeder can be controlled by both projects. However, it is highly recommended that Join 5 in the Aquarium project is disabled when the FishFeeder Project is executed.

All fish sensor and actuator device models (Figure 8 (3)-(5) and Figure 10 (2)-(6)) and the Join functions were developed in FishTalk. The video monitoring/control and dashboard (Figure 6) were developed in FishTalk and then became a general monitor & control cyber IoT device accommodated in IoTtalk for other IoTtalk applications. The Messaging device model (Figure 10 (7)) is reused from IoTtalk.

VI. PERFORMANCE EVALUATION

In an IoT system, the delays of messages delivered from the sensors to the actuators significantly impact the execution of the IoT applications. If the delays are too long, the system may fail to carry out the desired tasks. Also, if the messages are lost, important information may be lost in FishTalk. This section investigates the effect of FishTalk message delays in two aspects. First, in an aquarium, the conditions of the water may change rapidly. Therefore, when FishTalk detects poor water condition, it should activate the actuators fast enough or the fish will be in danger. Second, the frequency of sending the IoT messages should be appropriately selected so that important information will not be lost, while the energy is not significantly consumed by frequent message delivery.

A. EFFECT OF MESSAGE DELAY

Let \( t_s \) be the delay of sending a measured data from a FishTalk sensor to the FishTalk server and \( t_a \) be the delay of sending an instruction from the FishTalk server to a FishTalk actuator. We assume both \( t_s \) and \( t_a \) to be random variables with the density functions \( f_s(t_s) \) and \( f_a(t_a) \), respectively. We have obtained the histograms for \( t_s \) and \( t_a \) through 1000 measurements for the NB-IoT transmission delays. From the measured samples \( t_{NS} \) and \( t_{NA} \) for NB-IoT transmission, we can approximate \( t_{NS} \) and \( t_{NA} \) by Erlang distributions with the expected value \( \text{E}[t_{NS}] = 0.3792s \) and the variance \( \text{V}[t_{NS}] = 0.5013 \text{E}[t_{NS}]^2 \), and \( \text{E}[t_{NA}] = 1.6461s \) and \( \text{V}[t_{NA}] = 0.1363\text{E}[t_{NA}]^2 \). In [20] we also measured \( t_{WS} \) and \( t_{WA} \), the \( t_s \) and the \( t_a \) delays for Wi-Fi, and found that \( t_{WS} \) and \( t_{WA} \) have the same distribution. Therefore, we denote \( t_{WS} = t_{WS} \) and \( t_{WA} \), where \( \text{E}[t_{WS}] = 31.68ms \) and \( \text{V}[t_{WS}] = 0.0022\text{E}[t_{WS}]^2 \). The Erlang density function with the shape parameter \( n \) and the scale parameter \( \lambda \) is

\[
f_E(t, n, \lambda) = \frac{n^t e^{-\lambda t}}{(n-1)!} \tag{1}
\]

and

\[
\int_0^\infty f_E(t, n, \lambda) dt = 1 - \sum_{j=0}^{n-1} \frac{\lambda^j t^j e^{-\lambda t}}{j!}
\]

where \( \text{E}[t] = \frac{n}{\lambda} \) and \( \text{V}[t] = \frac{n}{\lambda^2} \).

FIGURE 12. The delay histograms.

From (1) we approximate \( f_s(t_{NS}) \) as \( f_E(t_{NS}, n_{NS}, \lambda_{NS}) \) where the shape parameter is \( n_{NS} = 2 \) an the scale parameter is \( \lambda_{NS} = 5.26 \). Similarly, we approximate \( f_a(t_{NA}) \) as \( f_E(t_{NA}, n_{NA}, \lambda_{NA}) \) where \( n_{NA} = 7 \) and \( \lambda_{NA} = 4.46 \). The approximations are validated by the Kolmogorov-Smirnov test for goodness of fit (see the solid and the dashed curves).

As we mentioned before, the delays \( t_s \) and \( t_a \) affect the control of the tank water environment. Consider the situation where the temperature of the tank water increases because the air conditioner in the room is turned off in a hot summer day or the heater was accidentally turned on (e.g., by the absent fish owner). Suppose that total ammonia in the water is 2 ppm, the pH level is 7.6, and we would like to keep the fish healthy. Therefore, when FishTalk detects poor water condition, it should activate the actuators fast enough or the fish will be in danger. Second, the frequency of sending the IoT messages should be appropriately selected so that important information will not be lost, while the energy is not significantly consumed by frequent message delivery.
affects the health of fish. Let \( \tau_f = \tau_{d,3} - \tau_{d,2} \). Then \( \tau_f \) is the period that the temperature increases from the caution point to the danger point. Let \( f_\tau(t_\tau) \) be the density function of \( t_\tau \). We have obtained the histogram for \( t_\tau \) through 1000 measurements for temperature changes due to heating. Denote the measured values as \( t_\tau^* \). From the measured samples, we can approximate \( t_\tau^* \) by the Gamma distribution with the expected value \( E[t_\tau] = 9228.319s \) and the variance \( V[t_\tau^2] = 0.0511 E[t_\tau^2] \). Therefore, \( f_\tau(t_\tau^*) \) has the shape parameter \( \alpha = 19.5752 \) and the scale parameter \( \beta = 0.0021 \).

\[
Pr[t_\tau > t_d] = 1 - \sum_{j=0}^{n_t+n_{r,a}-1} \binom{\alpha+j-1}{j} \left( \frac{\lambda^j \beta^\alpha}{(\lambda+\beta)^{\alpha+j}} \right)
\]

We have implemented simulation to compare with the analytic model (the \( Pr[t_\tau > t_d] \) equation above) with various values for input parameters of \( t_\tau \) and \( t_d \), and the error is less than 0.07%. Therefore, both the simulation and the analytic models are correct. Then we use the measured data to compute \( Pr[t_\tau^* > t_d^*] \) for NB-IoT (i.e., \( t_d^* = t_{N,d} + t_{n,a} \)) and Wi-Fi (i.e., \( t_d^* = t_{W,s} + t_{w,a} \)). We obtained the results \( Pr[t_\tau^* > t_d^*] \approx 1 \) for both B-IoT and Wi-Fi. That is, in the existing design of FishTalk, the message delays are short enough, and FishTalk always activates the actuators fast enough before the fish are in danger. Now we consider the cases when the communications system is not well designed such that \( t_\alpha, t_\beta \) and their variances are very large, i.e., \( E[t_d] \gg E[t_d^*] \) and \( V[t_d] \gg V[t_d^*] \). Without loss of generality, we increase \( E[t_d] \) and \( V[t_d] \) to see how it affects \( Pr[t_\tau^* > t_d] \). Figure 14 indicates that as \( E[t_d] \) and \( V[t_d] \) increase, \( Pr[t_\tau^* > t_d] \) decreases. When \( V[t_d^*] < 1000 \) \( V[t_d^*] \) or \( V[t_d] < 1000 \) \( V[t_d^*] \), \( Pr[t_\tau^* > t_d] \approx 1 \). When \( V[t_d] > 10000 \) \( V[t_d^*] \) and \( V[t_d^*] > 10000 \) \( V[t_d^*] \), \( Pr[t_\tau^* > t_d] \) drops. Figure 14 shows that FishTalk operates well for a communication system with large delays.

It is important that FishTalk guarantees that \( \tau_{d,3} > \tau_{d,2} \) so that the tank water does not reach the danger point. In Appendix A we derive the probability \( Pr[\tau_{d,3} > \tau_{d,2}] \) as follows.

\[
Pr[t_\tau > t_d] = 1 - \sum_{j=0}^{n_t+n_{r,a}-1} \binom{\alpha+j-1}{j} \left( \frac{\lambda^j \beta^\alpha}{(\lambda+\beta)^{\alpha+j}} \right)
\]

We have implemented simulation to compare with the analytic model (the \( Pr[t_\tau > t_d] \) equation above) with various values for input parameters of \( t_\tau \) and \( t_d \), and the error is less than 0.07%. Therefore, both the simulation and the analytic models are correct. Then we use the measured data to compute \( Pr[t_\tau^* > t_d^*] \) for NB-IoT (i.e., \( t_d^* = t_{N,d} + t_{n,a} \)) and Wi-Fi (i.e., \( t_d^* = t_{W,s} + t_{w,a} \)). We obtained the results \( Pr[t_\tau^* > t_d^*] \approx 1 \) for both B-IoT and Wi-Fi. That is, in the existing design of FishTalk, the message delays are short enough, and FishTalk always activates the actuators fast enough before the fish are in danger. Now we consider the cases when the communications system is not well designed such that \( t_\alpha, t_\beta \) and their variances are very large, i.e., \( E[t_d] \gg E[t_d^*] \) and \( V[t_d] \gg V[t_d^*] \). Without loss of generality, we increase \( E[t_d] \) and \( V[t_d] \) to see how it affects \( Pr[t_\tau^* > t_d] \). Figure 14 indicates that as \( E[t_d] \) and \( V[t_d] \) increase, \( Pr[t_\tau^* > t_d] \) decreases. When \( V[t_d^*] < 1000 \) \( V[t_d^*] \) or \( V[t_d] < 1000 \) \( V[t_d^*] \), \( Pr[t_\tau^* > t_d] \approx 1 \). When \( V[t_d] > 10000 \) \( V[t_d^*] \) and \( V[t_d^*] > 10000 \) \( V[t_d^*] \), \( Pr[t_\tau^* > t_d] \) drops. Figure 14 shows that FishTalk operates well for a communication system with large delays.

\[
Pr[t_\tau > t_d] = \sum_{j=0}^{n_t+n_{r,a}-1} \binom{\alpha+j-1}{j} \left( \frac{\lambda^j \beta^\alpha}{(\lambda+\beta)^{\alpha+j}} \right)
\]

We have implemented simulation to compare with the analytic model (the \( Pr[t_\tau > t_d] \) equation above) with various values for input parameters of \( t_\tau \) and \( t_d \), and the errors are less than 0.1%. Therefore, both the simulation and the analytic models are correct. Then we compute \( Pr[t_\tau > t_d] \) with the measured data \( t_\tau^* \) and different \( t_d \) values. We consider fixed report intervals \( t_{r,i} = t_{m,i} - t_{m,i-1} \) be i.i.d. random variables with the density function \( f_m(t_{m,i}) \) and the Laplace transform \( f_m^*(s) \). Let \( T_{m,i} = t_{m,i} + t_{m,2} + \cdots + t_{m,i} \). Then \( t_{m,0} + T_{m,i} \) is the time when the FishTalk server hears from the sensor. If the water conditions become fatal to the fish in \( T_{m,i} \) before FishTalk takes any action (e.g., \( T_{m,i} > t_\tau \)), then the fish are in danger. On the other hand, we do not want to alert the fish owner immediately when a message is lost for two reasons. First, the lost message may not mean that the fish is in danger and should not bother the fish owner. Second, frequently alert messages will consume energy. Therefore, it is important to set the frequency of periodic sensing and when to alert the fish owner. In Appendix B, we derive the probability \( Pr[t_\tau > T_{m,i}] \) as

\[
Pr[t_\tau > T_{m,i}] = \sum_{j=0}^{n_t+n_{r,a}-1} \binom{\alpha+j-1}{j} \left( \frac{\lambda^j \beta^\alpha}{(\lambda+\beta)^{\alpha+j}} \right) \frac{(\beta j!)}{(\lambda j!)} \frac{(f_m^*(s)j!)}{(f_m^*(s)j!)}
\]

For \( 1 \leq i \leq I \), let \( t_{m,i} = t_{m,i} - t_{m,i-1} \) be i.i.d. random variables with the density function \( f_m(t_{m,i}) \) and the Laplace transform \( f_m^*(s) \). Let \( T_{m,i} = t_{m,i} + t_{m,2} + \cdots + t_{m,i} \). Then \( t_{m,0} + T_{m,i} \) is the time when the FishTalk server hears from the sensor. If the water conditions become fatal to the fish in \( T_{m,i} \) before FishTalk takes any action (e.g., \( T_{m,i} > t_\tau \)), then the fish are in danger. On the other hand, we do not want to alert the fish owner immediately when a message is lost for two reasons. First, the lost message may not mean that the fish is in danger and should not bother the fish owner. Second, frequently alert messages will consume energy. Therefore, it is important to set the frequency of periodic sensing and when to alert the fish owner. In Appendix B, we derive the probability \( Pr[t_\tau > T_{m,i}] \) as

\[
Pr[t_\tau > T_{m,i}] = \sum_{j=0}^{n_t+n_{r,a}-1} \binom{\alpha+j-1}{j} \left( \frac{\lambda^j \beta^\alpha}{(\lambda+\beta)^{\alpha+j}} \right) \frac{(\beta j!)}{(\lambda j!)} \frac{(f_m^*(s)j!)}{(f_m^*(s)j!)}
\]
VII. CONCLUSIONS

Based on an IoT solution called IoTtalk, this paper proposed the FishTalk system that allows the aquarium sensors to drive the actuators in real time. We have made the following contributions not found in the previous approaches [3],[4],[5],[6],[7]:

- We clearly described the relationship between aquarium sensors and the actuators, and gave concrete examples about threshold setting that can be easily translated for programming in an IoT system.
- We showed how to intelligently feed the fish through a combined automate and manual control mechanism such that the fish are neither over nor under fed, and at the same time, the fish owner can enjoy watching fish feeding remotely. Our solution allows the designer to quickly deploy intelligent control for various water conditions.
- We have developed analytic model, simulation and measurement experiments to investigate the effects of IoT message delays and loss on the water condition control. Such study has not been conducted by existing smart aquarium solutions.

Although FishTalk aims for mini aquarium systems, the software can be directly reused in a large-scale fish farm after appropriate replacement of the actuators and parameter setting.

As a final remark, we briefly discuss the cost incurred to develop a FishTalk aquarium. The costs are calculated as follows.

1) Sensors (Figure 5 (1)) and actuators (Figure 5 (2)): FishTalk accommodates off-the-shelf sensors and actuators. Therefore, the customers can choose the sensors/actuators with the prices that fit their budget constraints. The sensor/actuator costs are the same as the previous solutions except that in FishTalk, some of the devices can be shared with other IoT applications that reuse the tank water (e.g., the smart plant applications). By doing so, the equipment costs for FishTalk and the smart plant application are actually reduced in the saloon of the NCTU student dormitory.

2) The control board (Figure 5 (4) and (6)): if we use ESP8266, the cost is about 4 USD. For other solutions in Table 1, the costs are higher.

3) The FishTalk server (Figure 5 (3)): it will become open source software for free.

4) Monitoring and Control (Figure 5 (7)): we use a smartphone with the web-based application for monitoring and control. We assume that everyone has a smartphone and there is no need to purchase one specifically for FishTalk. The web-based software is free. If a camera is included (the fish owner may also use it for home security or other smart applications), we can purchase any off-the-shelf camera within the allowable budget, and the cost is shared by all IoT applications in the room.

APPENDIX A

This appendix derives the probability $Pr[t_{d,3} > t_{d,2}]$ as follows. It is clear that $Pr[t_{d,3} > t_{d,2}] = Pr[t_T > t_s + t_a]$, and

$$Pr[t_T > t_s + t_a] = \int_{t_T=0}^{\infty} \int_{t_s=0}^{t_T} f_T(t_T) f_E(t_a, n_a, \lambda_a) f_E(t_s, n_s, \lambda_s) dt_a dt_s dt_T$$

$$= 1 - \sum_{j=0}^{n_s-1} \frac{\lambda_s}{j!} \sum_{i=0}^{j} \frac{(-1)^i (t_s)^i}{i!} \lambda_s^j t_T^{j-i}$$

$$= 1 - \sum_{j=0}^{n_s-1} \frac{\lambda_s}{j!} \sum_{i=0}^{j} \frac{(-1)^i (t_s)^i}{i!} \lambda_s^j t_T^{j-i} e^{-\lambda_s t_T}$$

Substitute (3) into (2) to yield

$$Pr[t_T > t_s + t_a] = \int_{t_T=0}^{\infty} \int_{t_s=0}^{t_T} f_T(t_T) f_E(t_a, n_a, \lambda_a) dt_a dt_T$$

$$- \int_{t_T=0}^{\infty} \int_{t_s=0}^{t_T} f_T(t_T) f_E(t_a, n_a, \lambda_a) \times \sum_{j=0}^{n_s-1} \frac{\lambda_s}{j!} \sum_{i=0}^{j} \frac{(-1)^i (t_s)^i}{i!} e^{-\lambda_s t_T} e^{\lambda_s t_a} dt_a dt_T$$

$$= A - B$$

where

$$A = \int_{t_T=0}^{\infty} \int_{t_s=0}^{t_T} f_T(t_T) f_E(t_a, n_a, \lambda_a) dt_a dt_T$$

and

$$B = \int_{t_T=0}^{\infty} \int_{t_s=0}^{t_T} f_T(t_T) f_E(t_a, n_a, \lambda_a)$$
\[
\times \left[ \sum_{j=0}^{n_s-1} \left( \frac{\lambda_a}{j!} \right) \sum_{i=0}^{j} \left( \frac{i}{j} \right) t_T^{-j-i} e^{-\lambda_s t_T(-t_a)^i} e^{\lambda_s t_a} \right] dt_a dt_T \quad (6)
\]

From (5) and (1), we have
\[
A = \int_{t_T=0}^{\infty} f_T(t_T) \left( 1 - \sum_{j=0}^{n_s-1} \frac{\lambda_a j}{j!} t_T^{-j} e^{-\lambda_s t_T} \right) dt_T
\]
\[
= 1 - \int_{t_T=0}^{\infty} f_T(t_T) \left( \sum_{j=0}^{n_s-1} \frac{\lambda_a j}{j!} t_T^{-j} e^{-\lambda_s t_T} \right) dt_T
\]
\[
= 1 - \sum_{j=0}^{n_s-1} \left( \frac{\lambda_a}{j!} \right) \int_{t_T=0}^{\infty} t_T^j f_T(t_T) e^{-\lambda_s t_T} dt_T \quad (7)
\]

From the frequency-domain general derivative of the Laplace transform, for a function \( f(t) \) with the Laplace transform \( f^*(s) \) we have
\[
\int_{t=0}^{\infty} t^i f(t) e^{-st} dt = (-1)^i \frac{d^i f^*(s)}{ds^i} \quad (8)
\]

From (8), (7) is rewritten as
\[
A = 1 - \sum_{j=0}^{n_s-1} \left( \frac{\lambda_a}{j!} \right) (-1)^j \left[ \frac{d^j f_T^*(s)}{ds^j} \right]_{s=\lambda_a}
\]
\[
= 1 - \sum_{j=0}^{n_s-1} \left[ \frac{(-\lambda_a)^j}{j!} \right] \left[ \frac{d^j f_T^*(s)}{ds^j} \right]_{s=\lambda_a} \quad (9)
\]

From (6),
\[
B = \sum_{j=0}^{n_s-1} \left( \frac{\lambda_a}{j!} \right) \sum_{i=0}^{j} \left( \frac{i}{j} \right) \int_{t_T=0}^{\infty} f_T(t_T) dt_T
\]
\[
\times \sum_{j=0}^{n_s-1} \left( \frac{\lambda_a}{j!} \right) \sum_{i=0}^{j} \left( \frac{i}{j} \right) (-1)^i \int_{t_T=0}^{\infty} f_T(t_T) t_T^{-j-i} e^{-\lambda_s t_T} dt_T
\]
\[
= \sum_{j=0}^{n_s-1} \left( \frac{\lambda_a}{j!} \right) \sum_{i=0}^{j} \left( \frac{i}{j} \right) (-1)^i \int_{t_T=0}^{\infty} f_T(t_T) t_T^{-j-i} e^{-\lambda_s t_T} dt_T
\]
\[
\times \sum_{j=0}^{n_s-1} \left( \frac{\lambda_a}{j!} \right) \sum_{i=0}^{j} \left( \frac{i}{j} \right) (-1)^i \int_{t_T=0}^{\infty} f_T(t_T) t_T^{-j-i} e^{-\lambda_s t_T} dt_T
\]
\[
\times \sum_{j=0}^{n_s-1} \left( \frac{\lambda_a}{j!} \right) \sum_{i=0}^{j} \left( \frac{i}{j} \right) (-1)^i \int_{t_T=0}^{\infty} f_T(t_T) t_T^{-j-i} e^{-\lambda_s t_T} dt_T \quad (10)
\]

In (10), we have
\[
\int_{t_T=0}^{\infty} f_T(t_T) t_T^{-j-i} e^{-\lambda_s t_T} dt_T
\]
\[
= \int_{t_T=0}^{\infty} \left[ \frac{\lambda_a}{j!} \right] t_T^{-j-i} e^{-\lambda_s t_T} dt_T
\]
\[
= \left[ \frac{\lambda_a}{j!} \right] t_T^{-j-i} e^{-\lambda_s t_T} dt_T
\]
\[
= \left[ \frac{\lambda_a}{j!} \right] \left[ \frac{(-\lambda_a)^j}{j!} \right] \left[ \frac{d^j f_T^*(s)}{ds^j} \right]_{s=\lambda_a}
\]
\[
= \left[ \frac{\lambda_a}{j!} \right] \left[ \frac{(-\lambda_a)^j}{j!} \right] \left[ \frac{d^j f_T^*(s)}{ds^j} \right]_{s=\lambda_a}
\]

From (12), (15) and (16), we have
\[
B = \sum_{j=0}^{n_s-1} \left( \frac{\lambda_a}{j!} \right) \sum_{i=0}^{j} \left( \frac{i}{j} \right) (-1)^i \int_{t_T=0}^{\infty} f_T(t_T) t_T^{-j-i} e^{-\lambda_s t_T} dt_T
\]
\[
\times \left[ \frac{\lambda_a}{j!} \right] \left[ \frac{(-\lambda_a)^j}{j!} \right] \left[ \frac{d^j f_T^*(s)}{ds^j} \right]_{s=\lambda_a}
\]
\[
= \sum_{j=0}^{n_s-1} \left( \frac{\lambda_a}{j!} \right) \sum_{i=0}^{j} \left( \frac{i}{j} \right) (-1)^i \left[ \frac{\lambda_a}{j!} \right] \left[ \frac{(-\lambda_a)^j}{j!} \right] \left[ \frac{d^j f_T^*(s)}{ds^j} \right]_{s=\lambda_a}
\]
\[
= \sum_{j=0}^{n_s-1} \left( \frac{\lambda_a}{j!} \right) \sum_{i=0}^{j} \left( \frac{i}{j} \right) (-1)^i \left[ \frac{\lambda_a}{j!} \right] \left[ \frac{(-\lambda_a)^j}{j!} \right] \left[ \frac{d^j f_T^*(s)}{ds^j} \right]_{s=\lambda_a}
\]

Equation (13) is rewritten as
\[
C = (-1)^{j-i} \left[ \frac{\lambda_a (n_a+i-1)!}{(\lambda_a-\lambda_s)^{n_a+i}} \right] \left[ \frac{d^{j-i} f_T^*(s)}{ds^{j-i}} \right]_{s=\lambda_a}
\]
\[
D = \sum_{k=0}^{n_a+i-1} \left[ \frac{\lambda_a}{k!} \right] \left[ \frac{d^k f_T^*(s)}{ds^k} \right]_{s=\lambda_a}
\]

From (12), (15) and (16), we have
\[
B = \sum_{j=0}^{n_s-1} \left[ \frac{\lambda_a}{j!} \right] \sum_{i=0}^{j} \left( \frac{i}{j} \right) \left[ \frac{\lambda_a}{j!} \right] \left[ \frac{(-\lambda_a)^j}{j!} \right] \left[ \frac{d^j f_T^*(s)}{ds^j} \right]_{s=\lambda_a}
\]
\[
\times \left[ \frac{\lambda_a}{j!} \right] \left[ \frac{(-\lambda_a)^j}{j!} \right] \left[ \frac{d^j f_T^*(s)}{ds^j} \right]_{s=\lambda_a}
\]
\[
= \sum_{j=0}^{n_s-1} \left[ \frac{\lambda_a}{j!} \right] \sum_{i=0}^{j} \left( \frac{i}{j} \right) \left[ \frac{\lambda_a}{j!} \right] \left[ \frac{(-\lambda_a)^j}{j!} \right] \left[ \frac{d^j f_T^*(s)}{ds^j} \right]_{s=\lambda_a}
\]
\[
= \sum_{j=0}^{n_s-1} \left[ \frac{\lambda_a}{j!} \right] \sum_{i=0}^{j} \left( \frac{i}{j} \right) \left[ \frac{\lambda_a}{j!} \right] \left[ \frac{(-\lambda_a)^j}{j!} \right] \left[ \frac{d^j f_T^*(s)}{ds^j} \right]_{s=\lambda_a}
\]
\[
\frac{1}{\lambda^j j!} \sum_{k=0}^{n_k} \frac{(-\lambda)^{n_k}}{k!} \left(\frac{\lambda}{\lambda - \lambda_a}\right)^{n_k} (j - i)! \left(\frac{-\lambda_a}{\lambda - \lambda_a}\right)^{na+i} (j - i)! \left(\frac{\lambda}{\lambda - \lambda_a}\right)^{na+i} (j - i)!
\]

\[
\sum_{j=0}^{n-a-1} \left(\frac{(-\lambda)^{j}}{j!} \right) \left(\frac{\lambda^{a+j}}{\lambda + \beta^{a+j}}\right)
\]

\[
\Pr\{t_T > T_m,I\} = \int_{t_T=0}^{\infty} f_T(t_T) f_{T_m,I}(t_T) dT_m dt_I
\]

\[
\Pr\{t_T > T_m,I\} = \int_{t_T=0}^{\infty} f_T(t_T) f_{T_m,I}(t_T) dT_m dt_I
\]


